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hence $b = \frac{5}{3}a^2$, $c = -\frac{1}{2}a^3$, $d = \frac{2}{3}a^4$, $e = \frac{8}{27}a^5$, and therefore

$$\sqrt[5]{\frac{3b}{5}} = -3\sqrt[3]{\frac{c}{10}} = 4\sqrt[4]{\frac{27d}{20}} = 5\sqrt[5]{\frac{27e}{8}}.$$

II. Solution by E. L. RICH, Lehigh University.

Consider the cube, $x^3 + 3px^2 + 3p^2x + x^3$, and the square, $x^2 + 2qx + q^2$. Their product is,

$$x^5 + (2q + 3p)x^4 + (q^2 + 3p^2 + 6pq)x^3 + (3pq^2 + 6p^2q + p^3)x^2 + (2p^3q + 3p^2q^2)x + p^3q^2.$$

Then the first condition is $2q + 3p = 0$ or $q = -\frac{2}{3}p$. The other conditions for the values of the coefficients gotten by equating coefficients, and substituting the first condition are,

$$b = \frac{1}{4}p^2, \quad c = -\frac{5}{4}p^3, \quad d = \frac{1}{4}p^4, \quad e = -\frac{3}{4}p^5.$$

Also solved by J. Scheffer, G. B. M. Zerr, Elmer Schuyler, and F. P. Matz.

218. Proposed by SAUL EPSTEIN, The University of Chicago, Chicago, Ill.

Prove that $\sum_{r=0}^n \frac{c_r}{r+1} = \frac{2^{n+1}-1}{n+1}$ where c_r is the coefficient of x^r in the expansion of $(1+x)^n$.

I. Solution by G. W. GREENWOOD, M. A. (Oxon), and HOWARD M. ARMSTRONG.

$$\sum_{r=0}^n \frac{c_r}{r+1} = \int_0^1 [c_0 + c_1x + c_2x^2 + \dots + c_nx^n] dx = \int_0^1 (1+x)^n dx = \frac{2^{n+1}-1}{n+1}.$$

II. Solution by J. SCHEFFER, Kee Mar College.

By the binomial theorem

$$2^{n+1}-1 = (1+1)^{n+1}-1 = (n+1) + \frac{(n+1)n}{1.2} + \frac{(n+1)n(n-1)}{1.2.3} + \dots$$

$$\therefore \frac{2^{n+1}-1}{n+1} = 1 + \frac{n}{1.2} + \frac{n(n-1)}{1.2.3} + \dots = 1 + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \dots = \sum_{r=0}^n \frac{c_r}{n+1}.$$

III. Solution by CHRISTIAN HORNUNG, Heidelberg University, Tiffin, Ohio.

$$\sum_{r=0}^n \frac{c_r}{r+1} = c_0 + \frac{1}{2}c_1 + \frac{1}{3}c_2 + \frac{1}{4}c_3 + \dots + \frac{1}{r+1}c_r + \frac{1}{n+1}c_n$$

$$= 1 + \frac{1}{2}n + \frac{1}{3} \cdot \frac{n(n-1)}{2!} + \frac{1}{4} \cdot \frac{n(n-1)(n-2)}{3!} + \dots + \frac{1}{n+1}$$

$$= \frac{1}{n+1} (n+1 + \frac{1}{2}n(n+1) + \frac{1}{3} \cdot \frac{n(n-1)(n+1)}{2!} + \dots + 1)$$

$$\begin{aligned}
 &= \frac{1}{n+1} \left(1+n+1 + \frac{n(n+1)}{2!} + \frac{n(n+1)(n-1)}{3!} + \dots + 1 \right) - 1 \\
 &= \frac{1}{n+1} (1+1)^{n+1} - 1 = \frac{2^{n+1}-1}{n+1}.
 \end{aligned}$$

Demonstrated by M. E. Graber by the method of mathematical induction, and by Elmer Schuyler by means of the integral calculus. Also solved by G. B. M. Zerr.

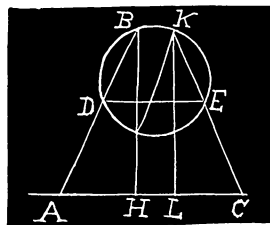
GEOMETRY.

236. Proposed by J. R. HITT, San Marcos, Texas.

If two sides of a triangle pass through fixed points, the third side touches a fixed circle.

I. Solution by the PROPOSER.

Let the sides AB , BC of triangle ABC pass through the points D , E , respectively. Draw BH perpendicular to AC , from F , intersection of circumcircle of triangle DBE with BH , draw diameter FK . Draw BK , KL perpendicular to AC . Arc DBE is locus of B . Since $\angle ABH$ is constant, F is a fixed point. $\therefore K$ is fixed. FBK is a right angle. $\therefore BHLK$ is a rectangle. $\therefore KL=BH$, a constant. Hence AC touches the circle with center K and radius KL .



II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W Va.

Let O , G be the two fixed points; a , b , c the sides of the triangle; $OG=h$, $OA=r$, $\angle AOX = \angle FOB = \theta$, $AD=p$. Then $AD:OE=c:c-r$.

$$\therefore OE = \frac{p}{c}(c-r).$$

$$\text{Also } \cos EOB = p/c, \text{ or } EOB = \cos^{-1}(p/c) = \beta.$$

$$\therefore p \cos(\phi + \beta - \pi - \theta) = \frac{p}{c}(c-r) \text{ is the equation to}$$

CB . Let $\phi + \beta - \pi = \delta$.

$$\therefore p \cos \delta \cos \theta + p \sin \delta \sin \theta = \frac{p}{c}(c-r).$$

$$AG = \sqrt{(h^2 + r^2 - 2rh \sin \theta)}.$$

$$\therefore h^2 = h^2 + r^2 - rh \sin \theta + r^2 - 2r \sqrt{(h^2 + r^2 - 2rh \sin \theta)} \cos A.$$

$$\therefore r^2 \sin^2 A - 2rh \sin \theta \sin^2 A = h^2 \cos^2 A - h^2 \sin^2 \theta.$$

$$\therefore \sin \theta = \frac{1}{h} [r \sin^2 A \pm \cos A \sqrt{(h^2 - r^2 \sin^2 A)}],$$

$$\cos \theta = \frac{\sin A}{h} [r \cos A \mp \sqrt{(h^2 - r^2 \sin^2 A)}].$$

$$\therefore \frac{pr}{h} \sin A \cos(\delta - A) + \frac{pr}{c} \cos p = + \frac{p}{h} \sqrt{(h^2 - r^2 \sin^2 A)} \sin(\delta - A).$$

